Calibration of Lens Distortion for Super-Wide-Angle Stereo Vision

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Abstract—This paper describes a novel calibration method for stereo cameras equipped with super-wide-angle low-distortion lenses. Radial distortion properties, which are difficult for polynomial distortion models to fit, are a common property of such lenses; this in turn affects camera parameter estimation, making more flexible models desirable. Therefore we employ a B-spline distortion model to properly fit radial distortion properties. Effectiveness of the model is characterized through experimental curve fitting to distortion layout data, camera calibration with regular lenses, and evaluation of 3D reconstruction.

I. INTRODUCTION

Stereo camera rigs and associated depth-estimation software have become popular as sensor solutions in robotics, due to their ability to collect both visual and depth data while working in either indoors or outdoors environments. Unfortunately, stereo-based depth estimation requires objects to be captured by both cameras simultaneously, so differences in apparent position relative to each camera (the disparity) can be measured. This restricts estimation to the intersection of the cameras’ fields of view (FOV’s). In applications where depth measurements over a wide area are required, this limitation can be mitigated by the employment of wide-angle lenses. However, such lenses generally display large distortions, which become harder to correct as intensity of distortion increases. Additionally, corrections to highly distorted images lead to partial reductions in image resolution.

In recent years, Nitto Kogaku K.K. and Theia Technologies LLC jointly developed a series of super-wide-angle, low-distortion camera lenses. These would seem to enable stereo camera rigs to have wide stereo FOV’s without incurring resolution losses. In fact, the lenses’ elaborate optical structures, which combine multiple optical elements, produce subtler but complex distortions. These cannot be properly addressed with calibration methods based on polynomial distortion models, commonly found in software packages such as OpenCV [1], which leads to failures in the calibration process.

This paper presents a camera calibration method based on a B-Spline distortion model, which is intended to address the limitations of current methods. Its effectiveness relative to polynomial distortion models is evaluated through experiments on curve fitting for distortion design of Theia lens. Moreover, experimental calibration and 3D reconstruction from images captured by a stereo camera equipped with Theia lenses were also performed.

II. RELATED WORK

Several calibration techniques exist for cameras with wide-angle or fisheye lenses. Devernay et al. [2] presented an automatic distortion calibration method that optimizes parameters of the distortion model based on the edge information in images. Thirthala et al. [3] introduced the radial trifocal tensor and proposed an approach that linearly computes the radial distortion of wide-angle lens. However, distortion models used in these researches to approximate the radial distortion are inflexible models such as polynomial model [4] or fisheye model [2].

To handle radial distortion effectively, we focused on the distortion model using B-spline function. Some previous researches deal with image distortion correction using B-spline function. Remy et al. [5] presented a distortion correction technique based on a B-spline distortion model, taking as inputs the position of the cameras’ optical axes and a set of captured images of a grid pattern. Jie-shou et al. [6] expressed the distortion of fisheye lenses using B-spline function and corrected the distortion without requiring knowledge on camera parameters such as optical axis. These researches lack comparison in performance with other distortion models, and depend on prior information such as the optical center. The method described in this paper differs from the previous ones in that it uses Zhang’s method [7] to compute the lenses’ distortion properties. We employed a B-spline distortion model as part of the projection model, and all camera parameters are estimated simultaneously. Additionally, we provide comparison of the B-spline distortion model and the traditional polynomial one.

III. LENS DISTORTION MODEL

When a lens is mounted on a camera, it usually causes the features of captured images to be bent or otherwise distorted. Such lens distortion can be more precisely defined as a deviation between the features of captured images and those predicted by the ideal pinhole camera projection model. Generally speaking, lens distortion effects become stronger as the camera FOV enabled by the lens becomes larger. Lenses composed of a single optical element tend to produce simple distortion effects. However, camera lenses...
are typically composed of multiple optical elements. By combining sets of complementing optical elements, a composite lens can be built so that both wide FOV and low distortion can be achieved. Unfortunately, what distortion does remain becomes all the more complex in nature, making approximation by simple models difficult. To deal with this problem, we employ a B-spline distortion model instead of the simpler polynomial model. In this section, we describe the traditional polynomial model and the proposed alternative B-spline model.

A. Brown’s Distortion Model

To incorporate lens distortion to iterative optimization, the distortion must be described by a distortion model. Brown’s distortion model [4] is one such model commonly used: it describes two major distortion factors, radial distortion and tangential distortion. Brown’s distortion model is described as

\[
\begin{align}
\begin{bmatrix}
x_d \\
y_d
\end{bmatrix} &= (1 + k_1 r^2 + k_2 r^4 + k_3 r^6 + \cdots) \begin{bmatrix}
x \\
y
\end{bmatrix} \\
&+ \begin{bmatrix}
2p_1 x y + p_2 (r^2 + 2x^2) \\
p_1 (r^2 + 2y^2) + 2p_2 x y
\end{bmatrix} (1 + p_3 r^2 + p_4 r^4 + \cdots),
\end{align}
\]

(1)

where \((x, y)\) is the ideal image point as projected on the image plane by ideal pin-hole projection; \(r = \sqrt{x^2 + y^2}\) is the distance between ideal image point and optical axis on the image plane; \(k_1, k_2, \ldots\) are the radial distortion coefficients; and \(p_1, p_2, \ldots\) are the tangential distortion coefficients. Each first and second terms from the right side of Eq. (1) describe radial distortion and tangential distortion by polynomial approximation. In most cases radial distortion is approximated up to the 6th order. For tangential distortion, coefficients \(p_1, p_2\) are often used.

B. B-spline Distortion Model

Instead of polynomial functions, B-spline functions can be used to express radial distortion. A B-spline function is a piecewise polynomial defined by pieces split at points known as knots. B-spline functions are continuous at the knots; when the knots are defined as \(q_0, q_1, \ldots, q_N\) \((q_0 < q_1 \leq \ldots \leq q_N)\), a B-spline function can be represented as follows:

\[
B_K(r) = \begin{cases} 
\alpha_{0,0} + \cdots + \alpha_{0,K-1} x^{K-1} & (q_0 \leq x < q_1) \\
\alpha_{1,0} + \cdots + \alpha_{1,K-1} x^{K-1} & (q_1 \leq x < q_2) \\
\vdots & \vdots \\
\alpha_{N-1,0} + \cdots + \alpha_{N-1,K-1} x^{K-1} & (q_{N-1} \leq x < q_N) 
\end{cases}
\]

(2)

where \(B_K(r)\) is a B-spline function composed of polynomials of degree \(K - 1\). B-spline function is computed as linear combination of B-spline basis by

\[
B_K(r) = \sum_{i=0}^{N-1} \alpha_i B_{i,K},
\]

(3)

where, \(B_{i,K}(r)\): a B-spline basis for \(i\)-th piece, \(\alpha_i\): a coefficient of linear combination for \(i\)-th piece. B-spline basis are determined uniquely by knots, \(\alpha\) and \(r\). B-spline basis can be computed by the De Boor-Cox calculation:

\[
B_{i,0}(r) = \begin{cases} 
1 & (q_i \leq r < q_{i+1}) \\
0 & (r < q_i, \ r \geq q_{i+1}) 
\end{cases},
\]

(4)

\[
B_{i,K}(r) = \frac{x - q_i}{q_{i+K} - q_i} B_{i,K-1}(r) + \frac{q_{i+K} - x}{q_{i+K} - q_i} B_{i+1,K-1}(r).
\]

(5)

From the above, a piecewise polynomial function of form Eq. (2) is computed.

The B-spline radial distortion function can thus be defined as:

\[
\begin{bmatrix}
x_d \\
y_d
\end{bmatrix} = \tilde{B}(x, y, \alpha, p)
\]

\[
= (1 + B_K(r)) \begin{bmatrix}
x \\
y
\end{bmatrix} + \begin{bmatrix}
2p_1 x y + p_2 (r^2 + 2x^2) \\
p_1 (r^2 + 2y^2) + 2p_2 x y
\end{bmatrix} (1 + p_3 r^2 + p_4 r^4 + \cdots).
\]

(6)

Here, \(\alpha\) is the vector of knot coefficients, \(p\) is the vector of tangential distortion parameters.

IV. SUPER-WIDE-ANGLE THEIA LENS

A series of camera lenses developed by Nitto Kogaku K.K. and Theia Technologies LLC are colloquially referred to as Theia lenses [8]. They are characterized by lens structures based on patented technologies, which enable super-wide FOV while keeping distortion low. Table I shows the specification for the MY125M model, which have the widest FOV of all Theia lenses; it is used throughout this paper as a prime example of a super-wide-angle, low-distortion lens.

Fig. 1 shows the radial distortion layout of MY125M as provided by the manufacturer. The plot shows the radial distortion effect by the lens as a function of the height of the projected image of a captured object, or imaged-height. Whereas radial distortion for a typical single lens follows a simple quadratic function, increasing as the square of distance from the center, the composite lens structure of MY125M produces a more complicated curve: distortion rate increases from ideal imaged-height 0 [mm] to nearly 1.5 [mm], at which point it starts to decrease.

V. CALIBRATION OF STEREO CAMERA

Reconstruction of 3D scene structure from stereo images requires knowledge of a number of camera parameters. Camera calibration is the process of estimating such parameters,
which can be divided into two categories: intrinsic parameters that describe translations between world coordinates and image coordinates, and extrinsic parameters that describe coordinate transformations between the two cameras.

In this paper, we use Zhang’s method [7], which works by computing an initial parameter estimation and then further optimizing it through an iterative method. The iterative method can deal with nonlinear distortion models such as the proposed B-spline model.

VI. CURVE FITTING BY B- SPLINE DISTORTION FUNCTION

In this section, we discuss curve fitting by a B-spline distortion function. Knot parameters on a free-knots B-spline function are difficult to optimize, therefore we resort to a fixed-knots B-spline function. Consequently, in the B-spline function, the parameters to be optimized are only \( \alpha_i \).

A. Fitting to Imaged-Height Dataset

Once radial distortion is assumed to be the only form of lens distortion, displacement by lens distortion can be described as change in imaged-height. If the data set of ideal imaged-height \( r_i \) and distorted imaged-height \( r'_i \) for \( i = 0, 1, \ldots, N - 1 \) is given, we can approximate the data set by the following B-spline model:

\[
r'_i = (1 + B_K(r)) r.
\]

(7)

The B-spline function is considered to satisfy the constraints of all ideal–distorted imaged-height pairs when the knots satisfy the Schoenberg-Whitney conditions:

\[
q_0 = q_1 = \cdots = q_{K-1} = r_0
\]

\[
q_j + K = \frac{r_j + r_{j+K}}{2} \quad (j = 0, 1, \ldots, N - K - 1)
\]

\[
q_N = q_{N+1} = \cdots = q_{N+K-1} = r_N.
\]

(8)

Under the Schoenberg-Whitney conditions, parameters \( \alpha_i \) can be derived by direct method:

\[
A \alpha = r',
\]

where,

\[
A = \begin{bmatrix}
B_{0,K}(r_0) & B_{1,K}(r_0) & \cdots & B_{N-1,K}(r_0) \\
B_{0,K}(r_1) & B_{1,K}(r_1) & \cdots & B_{N-1,K}(r_1) \\
\vdots & \vdots & \ddots & \vdots \\
B_{0,K}(r_{N-1}) & B_{1,K}(r_{N-1}) & \cdots & B_{N-1,K}(r_{N-1})
\end{bmatrix},
\]

\[
\alpha = \begin{bmatrix}
\alpha_0 \\
\alpha_1 \\
\vdots \\
\alpha_{N-1}
\end{bmatrix},
\]

\[
r' = \begin{bmatrix}
\frac{r'_{0}}{r_0} - 1 \\
\frac{r'_{1}}{r_1} - 1 \\
\vdots \\
\frac{r'_{N-1}}{r_{N-1}} - 1
\end{bmatrix}.
\]

(9)

B. Applying to Camera Calibration

For camera calibration, we use Zhang’s method with B-spline distortion model. It requires that a number of images of a planar pattern be captured by the camera, and multiple feature points be identified on each pattern image. Using a B-spline model to represent distortion in the reprojection function, parameters are computed by minimizing the following functional:

\[
\sum_{j} \sum_{i} \|x_i - \tilde{m}(X_{i,j}, M, \alpha, p, R_j, T_j)\|^2,
\]

(10)

where \( i = 0, \ldots, N \) is the number of feature points on planar pattern; \( j = 0, \ldots, L \) is the number of taken images; \( \tilde{m}(X_{i,j}, M, \alpha, p, R_j, T_j) \) is the reprojection function; \( X_i = [X_i, Y_i, 0]^T \) are the coordinates of a feature point on the planar pattern; \( x_i = [x_i, y_i]^T \) are the coordinates of a detected feature point (corresponding to \( X_i \)) on the image plane; \( M \) is the camera matrix; \( R_j, T_j \) are rotation and translation from planar object coordinates to image plane in the \( j \)th image; \( \alpha \) is the vector of coefficients for the B-spline function in the B-spline distortion model; and \( p \) is the vector of tangential distortion coefficients in the B-spline distortion model. The reprojection function computes coordinates of reprojected points \( (x'_i, y'_i) \) by the above parameters:

\[
\begin{bmatrix}
x'_i \\
y'_i
\end{bmatrix} = \tilde{m}(X_i, M, \alpha, p, R, T).
\]

(11)

The calculation of the reprojection function is defined as follows:

\[
\begin{bmatrix}
\tilde{x} \\
\tilde{y} \\
1
\end{bmatrix} = \begin{bmatrix} R & T \\ 0 & 1 \end{bmatrix} \begin{bmatrix} X_i \\ Y_i \\ 0 \\ 1 \end{bmatrix}
\]

(12)

\[
\begin{bmatrix}
x_d \\
y_d
\end{bmatrix} = \hat{B}(x, \tilde{y}, \alpha, p)
\]

(13)

\[
\tilde{m}(X_i, M, \alpha, p, R, T) = M \begin{bmatrix}
x_d \\
y_d \\
1
\end{bmatrix},
\]

(14)

where,

\[
M = \begin{bmatrix}
f_x & 0 & c_x \\
f_y & 0 & c_y \\
0 & 0 & 1
\end{bmatrix}.
\]

(15)
\( f_x, f_y \) are focal lengths for horizontal and vertical direction, 
\( c_x, c_y \) denote location of the optical center on the image plane. Determining the parameter values on Eq. (10) is a non-linear least squares problem, therefore we can solve it using the Levenberg-Marquardt algorithm. 

**VII. EXPERIMENTAL RESULT**

This section provides experimental results to validate the effectiveness of using a B-spline distortion model for calibrating a stereo rig equipped with super-wide-angle, low-distortion lenses such as the MY125M.

**A. Curve Fitting to Distortion Design Data of MY125M**

A B-spline distortion model and two comparison polynomial distortion models are fitted to the data set in Fig. 1. For the B-spline distortion model, the method described in Sec. VI-A is used. The degree of piecewise polynomials in the B-spline is 3, and the knots are 35 points determined by the data set and Eq. (8). The two polynomial distortion models are:

\[
\begin{align*}
    r' &= \left(1 + k_1 r^2 + k_2 r^4 + k_3 r^6\right) r, \\
    r' &= \left(1 + k_1 r^2 + k_2 r^4 + k_3 r^6 + k_4 r^8 + k_5 r^{10} + k_6 r^{12}\right) r.
\end{align*}
\]

One is a 6\(^{th}\) degree polynomial (commonly used in some major tools) and the other is a 12\(^{th}\) degree polynomial (for increased approximation accuracy). Curve fitting of these models is done by Levenberg-Marquardt algorithm.

The results are shown in Fig. 2, Fig. 3, and Fig. 4. Additionally, fitting errors for each model are shown in Fig. 5. For the polynomial model of 6\(^{th}\) degree, there several large errors, up to 2 [\(\mu m\)]. These errors decrease to about one-tenth for the polynomial model of 12\(^{th}\) degree. For the B-spline model, the curve correctly matches all data points.

**B. Camera Calibration of the Stereo Camera with MY125M**

In addition to testing our method against its radial distortion layout, we also performed camera calibration on a real stereo rig equipped with the MY125M. The stereo rig was a parallel stereo camera of baseline length 0.3 [m] composed of two PointGrey Flea2 FL2-08S2C color firewire cameras. Image resolution was set to 1024 \(\times\) 768 [pixel]. To set the depth of field to beyond 0.3 [m], lens focus was adjusted to 0.5 [m] destination. A calibration board consisted of a checker pattern printed on paper and glued on a resin plate as created and used as the planar pattern. Intersection points in the checker pattern are used as feature points. These are arranged in a 13 \(\times\) 9 grid, with a constant gap of 0.02 [mm] between them. By taking images of the calibration board by left and right cameras, the image sets shown in Fig. 6 and Fig. 7 were created. Parameter estimation was done by the method described in VI-B. The B-spline distortion model of Eq. (6) as well as two comparison polynomial distortion models were used. The knots of B-spline were the same as Sec. VII-A. For parameters of the tangential distortion, we use only \(p_1, p_2\). The following polynomial distortion models

Fig. 2. Fitting of 6\(^{th}\) degree polynomial model to radial distortion layout of MY125M.

Fig. 3. Fitting of 12\(^{th}\) degree polynomial model to radial distortion layout of MY125M.

Fig. 4. Fitting of 3\(^{rd}\) degree (cubic) B-spline model to radial distortion layout of MY125M.

Fig. 5. Fitting error for each distortion model.
were used:
\[
\begin{bmatrix}
x_d \\
y_d
\end{bmatrix} = (1 + k_1r^2 + k_2r^4 + k_3r^6) \begin{bmatrix}
x \\
y
\end{bmatrix}
+ 2p_1\tilde{x}\tilde{y} + p_2(r^2 + 2\tilde{x}^2) \\
p_1(r^2 + 2\tilde{y}^2) + 2p_2\tilde{x}\tilde{y}
\] \quad (18)

\[
\begin{bmatrix}
x_d \\
y_d
\end{bmatrix} = (1 + k_1r^2 + k_2r^4 + \ldots + k_6r^{12}) \begin{bmatrix}
x \\
y
\end{bmatrix}
+ 2p_1\tilde{x}\tilde{y} + p_2(r^2 + 2\tilde{x}^2) \\
p_1(r^2 + 2\tilde{y}^2) + 2p_2\tilde{x}\tilde{y}
\] \quad (19)

Final root mean square errors (RMSE) for all feature points are shown in Table II. The final RMSE for the B-spline distortion model is the smallest for both cameras. The curves of radial distortion rate are shown in Fig. 8 and Fig. 9. Here, we drew the layout curve as an indicator in those figures, although adjustment of the focus of lenses changes the real curves away from the layout curve. Theoretically, we expect the three distortion curves obtained by calibration to be almost the same. However, in Fig. 8 and Fig. 9, the actual curves significantly differ from each other. Since we simultaneously estimated parameters of the total projection model using Zhang’s method, distortion curves we suppose the difference of the distortion model affects other parameters (e.g. focal length, optical center). Additionally, because the RMSE of the B-spline distortion model is the smallest of all three models, we can conclude the curve computed from the B-spline distortion model best approximates the lenses’ real distortion properties.

Incidentally, between ideal imaged-height 2.5 [mm] and 3.0 [mm], the curves of some cameras and models depart from the outline of layout curve. We have speculated that this was caused by a lack of corresponding feature points in the image set, because the area which corresponds to those ideal imaged-height is relatively narrow on the CCD tip. Therefore, there are different trends between RMSE of both cameras, RMSE of 6th degree polynomial is larger than that of 12th one on left camera, it is opposite on right camera.

We considered that differences in individual lenses affected the suitability of the models and appeared as these trends.

<table>
<thead>
<tr>
<th>Distortion model</th>
<th>RMSE (left)</th>
<th>RMSE (right)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6th degree polynomial</td>
<td>0.183 [pixel]</td>
<td>0.223 [pixel]</td>
</tr>
<tr>
<td>12th degree polynomial</td>
<td>0.183 [pixel]</td>
<td>0.222 [pixel]</td>
</tr>
<tr>
<td>3rd degree B-spline</td>
<td>0.167 [pixel]</td>
<td>0.204 [pixel]</td>
</tr>
</tbody>
</table>
Fig. 10. Stereo benchmark images.

Fig. 11. 3D reconstruction with $3^{rd}$ degree B-spline distortion model (3D points in orthogonal projection).

C. 3D Reconstruction by Stereo Vision Processing

The parameters obtained in the same way as the experiments described in Sec. VII-B were applied to stereo vision processing. The extrinsic parameters required for stereo vision processing were computed from a separate image set composed of 40 stereo image pairs, all of which depict the same calibration board used in VII-A. In order to evaluate the accuracy of 3D reconstructions, a set of random dot patterns of dimensions 4.2 [m]×0.3 [m] (as shown in Fig. 10) were printed and pasted to a wall. The stereo camera was then placed at a point 1 [m] distant from the wall, with the frontal direction almost vertical to the wall. Under the assumption of correct calibration, depth estimation algorithms working on images captured under this setup should “see” an illusory 3D shape on the otherwise flat wall. A traditional block-matching algorithm was used to compute stereo correspondence on the captured images. The window size of block-matching was set to 33 [pixel]×33 [pixel].

The reconstructed 3D shapes from B-spline distortion model is shown in Fig. 11 and Fig. 12. Fig. 11 is front view of 3D shape, and Fig. 12 is horizontal projection view. For B-spline distortion model, the 3D shape appears at nearly 1 [m] depth in the correct scale. In contrast, for polynomial distortion models, we could not reconstruct enough 3D points to display the 3D shape. We concluded that the effect of the remaining distortion caused us to compute the wrong extrinsic parameters and 3D shape.

VIII. CONCLUSION

In this paper, a camera calibration method using a B-spline distortion model was presented as an appropriate method for calibration of stereo cameras equipped with super-wide-angle and low-distortion lenses. Experimental curve fitting to distortion layouts for Theia lens MY125M, camera calibration with MY125M and 3D reconstruction were performed. Through these experiments, the effectiveness of a B-spline distortion model relative polynomial models was demonstrated. However, distortion model fitting shows unfavorable results at the periphery of the visual field. This is conjectured to be due to a lack of feature points at the periphery of the images in the set used for camera calibration. Furthermore, this paper dealt with comparison of performance only between B-spline model and polynomial model. Since several other distortion models exist, we intend to compare B-spline model to those models.

Future work should therefore consider ways to address these issues.

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REFERENCES